Solutions to Mega HW #4
ECNS 561 (Fall 2015)
Due: 12/4/2015

1.)

4.6 (i) With $df = n - 2 = 86$, we obtain the 5% critical value from Table G.2 with $df = 90$. Because each test is two-tailed, the critical value is 1.987. The t statistic for H_0: $\beta_0 = 0$ is about -.89, which is much less than 1.987 in absolute value. Therefore, we fail to reject $\beta_0 = 0$. The t statistic for H_0: $\beta_1 = 1$ is $(.976 - 1)/.049 \approx -.49$, which is even less significant. (Remember, we reject H_0 in favor of H_1 in this case only if $|t| > 1.987$.)

(ii) We use the SSR form of the F statistic. We are testing $q = 2$ restrictions and the df in the unrestricted model is 86. We are given $SSR_r = 209{,}448.99$ and $SSR_{ur} = 165{,}644.51$. Therefore,

$$F = \frac{(209,448.99 - 165,644.51)}{165,644.51} \cdot \left(\frac{86}{2}\right) \approx 11.37,$$

which is a strong rejection of H_0: from Table G.3c, the 1% critical value with 2 and 90 df is 4.85.

(iii) We use the R-squared form of the F statistic. We are testing $q = 3$ restrictions and there are $88 - 5 = 83$ df in the unrestricted model. The F statistic is $[(.829 - .820)/(1 - .829)](83/3) \approx 1.46$. The 10% critical value (again using 90 denominator df in Table G.3a) is 2.15, so we fail to reject H_0 at even the 10% level. In fact, the p-value is about .23.

(iv) If heteroskedasticity were present, Assumption MLR.5 would be violated, and the F statistic would not have an F distribution under the null hypothesis. Therefore, comparing the F statistic against the usual critical values, or obtaining the p-value from the F distribution, would not be especially meaningful.

4.8 (i) We use Property VAR.3 from Appendix B: $\text{Var}(\hat{\beta}_1 - 3\hat{\beta}_2) = \text{Var}(\hat{\beta}_1) + 9\,\text{Var}(\hat{\beta}_2) - 6\,\text{Cov}(\hat{\beta}_1, \hat{\beta}_2)$.

(ii) $t = (\hat{\beta}_1 - 3\hat{\beta}_2 - 1)/\text{se}(\hat{\beta}_1 - 3\hat{\beta}_2)$, so we need the standard error of $\hat{\beta}_1 - 3\hat{\beta}_2$.

(iii) Because $\theta_1 = \beta_1 - 3\beta_2$, we can write $\beta_1 = \theta_1 + 3\beta_2$. Plugging this into the population model gives

$$\begin{aligned} y &= \beta_0 + (\theta_1 + 3\beta_2)x_1 + \beta_2 x_2 + \beta_3 x_3 + u \\ &= \beta_0 + \theta_1 x_1 + \beta_2(3x_1 + x_2) + \beta_3 x_3 + u. \end{aligned}$$

This last equation is what we would estimate by regressing y on x_1, $3x_1 + x_2$, and x_3. The coefficient and standard error on x_1 are what we want.

4.9 (i) With *df* = 706 – 4 = 702, we use the standard normal critical value (*df* = ∞ in Table G.2), which is 1.96 for a two-tailed test at the 5% level. Now $t_{educ} = -11.13/5.88 \approx -1.89$, so $|t_{educ}| = 1.89 < 1.96$, and we fail to reject H_0: $\beta_{educ} = 0$ at the 5% level. Also, $t_{age} \approx 1.52$, so *age* is also statistically insignificant at the 5% level.

(ii) We need to compute the *R*-squared form of the *F* statistic for joint significance. But $F = [(.113 - .103)/(1 - .113)](702/2) \approx 3.96$. The 5% critical value in the $F_{2,702}$ distribution can be obtained from Table G.3b with denominator *df* = ∞: *cv* = 3.00. Therefore, *educ* and *age* are jointly significant at the 5% level (3.96 > 3.00). In fact, the *p*-value is about .019, and so *educ* and *age* are jointly significant at the 2% level.

(iii) Not really. These variables are jointly significant, but including them only changes the coefficient on *totwrk* from –.151 to –.148.

(iv) The standard *t* and *F* statistics that we used assume homoskedasticity, in addition to the other CLM assumptions. If there is heteroskedasticity in the equation, the tests are no longer valid.

4.11 (i) In columns (2) and (3), the coefficient on *profmarg* is actually negative, although its *t* statistic is only about –1. It appears that, once firm sales and market value have been controlled for, profit margin has no effect on CEO salary.

(ii) We use column (3), which controls for the most factors affecting salary. The *t* statistic on log(*mktval*) is about 2.05, which is just significant at the 5% level against a two-sided alternative. (We can use the standard normal critical value, 1.96.) So log(*mktval*) is statistically significant. Because the coefficient is an elasticity, a ceteris paribus 10% increase in market value is predicted to increase *salary* by 1%. This is not a huge effect, but it is not negligible, either.

(iii) These variables are individually significant at low significance levels, with $t_{ceoten} \approx 3.11$ and $t_{comten} \approx -2.79$. Other factors fixed, another year as CEO with the company increases salary by about 1.71%. On the other hand, another year with the company, but not as CEO, lowers salary by about .92%. This second finding at first seems surprising, but could be related to the "superstar" effect: firms that hire CEOs from outside the company often go after a small pool of highly regarded candidates, and salaries of these people are bid up. More non-CEO years with a company makes it less likely the person was hired as an outside superstar.

4.12 (i) If *expend* increases by 10% then *lexpend* increases by .10. Multiplying .10 by 11.16 gives 1.116, or about 1.1 percentage points. So a 10% increase in spending is associated with a 1.1 percentage point increase in the math pass rate.

(ii) The low *R*-squared does not imply that *lexpend* is uncorrelated with the underlying error term, *u*, although it does show that there are many omitted factors that help explain *math*10. Even if the expenditure levels were randomly assigned to schools it is unlikely that the amount of variation in *math*10 explained by spending could be more than a few percent of the total variation.

(iii) The coefficient on *lexpend* falls to 7.75, but its *t* statistic is 2.55, which is statistically significant even at the 1% level. Presumably correlation between *lspend* and *lunch* make the simple regression estimates unreliable.

(iv) Adding the *lenroll* and *lnchprg* (essentially the poverty rate) allows us to explain notably more variation in *math*10, but it is still only about 19%. One might improve the explanatory power by including variables such as average family income and measures of parental education levels. Or, features of the teachers, such as the percentage having master's degrees, might help. A variable likely to be very predictive is a variable that, say, measures the pass rate on a 9th grade math test given in the previous year. But we might not want to hold such a variable fixed when studying the effects of spending.

2.)

5.1 Write $y = \beta_0 + \beta_1 x_1 + u$, and take the expected value: $E(y) = \beta_0 + \beta_1 E(x_1) + E(u)$, or $\mu_y = \beta_0 + \beta_1 \mu_x$ since $E(u) = 0$, where $\mu_y = E(y)$ and $\mu_x = E(x_1)$. We can rewrite this as $\beta_0 = \mu_y - \beta_1 \mu_x$. Now, $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}_1$. Taking the plim of this we have $\text{plim}(\hat{\beta}_0) = \text{plim}(\bar{y} - \hat{\beta}_1 \bar{x}_1) = \text{plim}(\bar{y}) - \text{plim}(\hat{\beta}_1)\cdot\text{plim}(\bar{x}_1) = \mu_y - \beta_1 \mu_x$, where we use the fact that $\text{plim}(\bar{y}) = \mu_y$ and $\text{plim}(\bar{x}_1) = \mu_x$ by the law of large numbers, and $\text{plim}(\hat{\beta}_1) = \beta_1$. We have also used the parts of Property PLIM.2 from Appendix C.

5.2 A higher tolerance of risk means more willingness to invest in the stock market, so $\beta_2 > 0$. By assumption, *funds* and *risktol* are positively correlated. Now we use equation (5.5), where $\delta_1 > 0$: $\text{plim}(\tilde{\beta}_1) = \beta_1 + \beta_2\delta_1 > \beta_1$, so $\tilde{\beta}_1$ has a positive inconsistency (asymptotic bias). This makes sense: if we omit *risktol* from the regression and it is positively correlated with *funds*, some of the estimated effect of *funds* is actually due to the effect of *risktol*.

5.4 Write y = $\beta_0 + \beta_1 x + u$, and take the expected value: E(y) = $\beta_0 + \beta_1$E(x) + E(u), or μ_y = $\beta_0 + \beta_1 \mu_x$, since E(u) = 0, where μ_y = E(y) and μ_x = E(x). We can rewrite this as $\beta_0 = \mu_y - \beta_1 \mu_x$. Now, $\tilde{\beta}_0 = \bar{y} - \tilde{\beta}_1 \bar{x}$. Taking the plim of this we have plim($\tilde{\beta}_0$) = plim($\bar{y} - \tilde{\beta}_1 \bar{x}$) = plim($\bar{y}$) – plim($\tilde{\beta}_1$)·plim($\bar{x}$) = $\mu_y - \beta_1 \mu_x$, where we use the fact that plim($\bar{y}$) = μ_y and plim($\bar{x}$) = μ_x by the law of large numbers, and plim($\tilde{\beta}_1$) = β_1. We have also used the parts of the Property PLIM.2 from Appendix C.

3.) (a) The t-statistic is $\frac{0.485}{2.61} = 0.186 < 1.96$. Therefore, the coefficient on BDR is not statistically significantly different from zero.

(b) The coefficient on *BDR* measures the *partial effect* of the number of bedrooms holding house size (*Hsize*) constant. Yet, the typical 5-bedroom house is much larger than the typical 2-bedroom house. Thus, the results in (a) says little about the conventional wisdom.

(c) The 99% confidence interval for effect of lot size on price is 2000 × [.002 ± 2.58 × .00048] or 1.52 to 6.48 (in thousands of dollars).

(d) Choosing the scale of the variables should be done to make the regression results easy to read and to interpret. If the lot size were measured in thousands of square feet, the estimate coefficient would be 2 instead of 0.002.

(e) The 10% critical value from the $F_{2,¥}$ distribution is 2.30. Because 0.08 < 2.30, the coefficients are not jointly significant at the 10% level.

4.) We can write e_i as $e_i = y_i - \hat{\boldsymbol{\beta}}'\boldsymbol{x}_i = (\boldsymbol{\beta}'\boldsymbol{x}_i + \varepsilon_i) - \hat{\boldsymbol{\beta}}'\boldsymbol{x}_i = \varepsilon_i + (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})'\boldsymbol{x}_i$
We know that plim$\hat{\boldsymbol{\beta}} = \boldsymbol{\beta}$ and x_i is unchanged as n approaches ∞. So, e_i is arbitrarily close to ε_i as n approaches ∞.

STATA Exercise

a.) The coefficient estimates for the unemployment rate and the voting ratio are statistically significant.

b.) We fail to reject the null that the coefficient on the unemployment rate is equal to zero and the coefficient on population density is zero. We conclude that these variables are jointly statistically significant.