ECNS 316
Game Theoretic Applications

Yezer (2014), Economics of Crime and Enforcement

Game Theory and Crime

- Game theory provides an alternative framework for studying the interactions between offenders, victims, and the justice system.
- Application of game theory can demonstrate how problems of enforcement and prison management can be addressed by modeling the strategic interaction of offenders and police. This can be done by relying on pure strategy equilibrium concepts.
- For many purposes, however, the strategic interaction between offenders and police results in a mixed-strategy equilibrium where both offending and enforcement are stochastic (i.e., randomly determined).

Definitions and Concepts

- **Players.** Agents who make choices (i.e., offenders, victims, or enforcers).
  - Behave rationally
  - Have strategic interactions where each player's welfare or payoffs depends on the choices made by other players
  - Players are aware, at least to an extent, of this strategic interaction
    - But, often lack full information regarding other player behavior
- **Actions.** Choices that can be made (e.g., decision to offend)
- **Rules, nature, information, and outcomes.** These compose the order in which decisions are made and the amount of information that each player has at each stage; they are vital for crime games

Definitions and Concepts

- **Payoffs.** Returns from different outcomes
  - E.g., expected gains, sanctions, losses, and costs of patrolling or other precautions
- **Strategy.** Rule for choosing actions
  - E.g., whether or not to take precautions, patrol, and offend
- **Strategy combination.** A strategy for each player. For instance, offend and not take precautions.
- **One-shot vs. repeated games.** We will focus only on one-shot games.
- **Equilibrium.** Solution concept for predicting strategy combinations that determine outcomes.

Equilibrium Concepts

- **Cooperative equilibrium**. This is the strategy combination that maximizes the joint return added over all players.
  - This could be achieved if players could make binding and enforceable contracts.
  - Crime, however, is one instance where cooperation is most unlikely because contracts cannot be enforced under the civil law
- **Dominant Strategy equilibrium.** There may be a single action giving the highest payoff to one player regardless of the actions chosen by the other players. When this is true for all players, there is a dominant strategy equilibrium.

Equilibrium Concepts

- **Nash equilibrium.** Given each player's choice, the other player's strategy choice is optimal.
  - Two types of Nash equilibria will be used in this lecture
    - 1.) A Nash equilibrium in pure strategies exists when there is a strategy combination consisting of single actions by each player such that no player can gain by changing her choice of actions.
      - There may be more than one Nash equilibrium in pure strategies in a game. In this case, each is a solution to the game.
      - Any dominant strategy equilibrium is also a Nash in pure strategies
    - 2.) A Nash equilibrium in mixed strategies occurs when players choose a strategy combination in which each player alternates between actions subject to a given probability distribution and in which, given the mixed strategy of one player, the mixed strategy of the other is optimal.
      - For example, player one may choose action A with probability = .6 and action B with probability = (1-.6) = .4, and player two may choose action C with probability = .3 and action D with probability = .7.
      - A mixed-strategy equilibrium exists if, given player one's probabilities of choosing A and B, player two has no incentive to change probabilities, and similarly player one has no incentive to change given player two's mixed strategy.

Prisoner's Dilemma

- Most basic and classic game theory application is the prisoner's dilemma
- Assume police have caught two suspects who have cooperated in commission of one or more crimes
- Standard practice is to separate suspects and interrogate individually while making the following offers:
  - 1.) Admit to the crime and serve as a witness against the other suspect, who has not confessed, and face a prison sentence of X months
  - 2.) Remain silent, and if other suspect confesses, face the regular prison sentence of 10X months
  - If both suspects confess, they will each serve sentences of 5X.
  - The suspects, in turn, realized that if no one confesses, there is a 30% chance they will be convicted anyway and receive a sentence of 10X months. So, if both fail to confess, their expected sentences are each $0.3(10X) = 3X$ months.

Prisoner's Dilemma Facing Two Suspects

		Alpha's Choices	
		Confess	Hold out
Beta's Choices	Confess	-5X, -5X	-X, -10X
	Hold out	-10X, -X	-3X, -3X

- Alpha chooses one of the two columns. Beta chooses one of the two rows
- Alpha's payoffs are shown on the right in each cell. Beta's payoffs are shown on the left.

- Q. What is the cooperative equilibrium?
- Ans. Hold out-Hold out. This maximizes their joint payoff.

- Q. What is the dominant strategy for each player?
- Ans. Consider Alpha. Conditional on Beta choosing "confess", Alpha will choose "confess" because -5X is a shorter sentence than -10X. Conditional on Beta choosing "hold out", Alpha will choose "confess" because –X is a shorter sentence than -3X. Thus, "confess" always has the higher payout and is a dominant strategy. Due to symmetry of the game, Beta's dominant strategy will be the same.

Prisoner's Dilemma Facing Two Suspects

Alpha's Choices

Beta's Choices

	Confess	Hold out
Confess	-5X, -5X	-X, -10X
Hold out	-10X, -X	-3X, -3X

- To find a Nash equilibrium in pure strategies, select any strategy combination and ask whether either player has an incentive to defect from that choice. If there is no incentive, it is a Nash in pure strategies.
- Q. Is there a Nash equilibrium for this game?
- Ans. In this case, the existence of a dominant strategy equilibrium implies there is a Nash in pure strategies at confess-confess.

- Here, the cooperative and Nash equilibriums did not align.
  - For the suspects, their welfare under the Nash was worse than under the cooperative equilibrium.
  - For those operating the criminal justice system, however, the outcome is perfectly satisfactory...this is why they separate the suspects and provide some relief from the harshest sentences in the cases in which both confess

Prisoner's Dilemma Facing Two Convicts

		Alpha's Choices	
		Cooperate	Inform
Beta's Choices	Cooperate	-5X, -5X	-15X, -2X
	Inform	-2X, -15X	-10X, -10X

- Another form of the prisoner's dilemma concerns cooperation among prison inmates.
- Assume Alpha and Beta have been convicted and are serving sentences of 10X years.
  - If they cooperate in an escape attempt, there is a 50% chance of success, and their expected time in prison would be 5X.
  - If one informs, then his sentence will be reduced to 2X and the other will be caught and have his sentence increased to 15X.
  - If both inform, there is no escape and no change in expected sentence.

- Q. What is the cooperative equilibrium?
- Ans. Cooperate-Cooperate gives the best joint payoff.

- Q. Is there a dominant strategy equilibrium?
- Ans. Yes! Inform-Inform is a dominant strategy equilibrium that is a Nash in pure strategies.

- Again, the Nash equilibrium is less attractive to the convicts than the cooperative equilibrium.

Prisoner's Dilemma Facing Two Convicts with Inmate Action Against Informers

Alpha's Choices

Beta's Choices		Cooperate	Inform
	Cooperate	-5X, -5X	-15X, -2X - Z
	Inform	-2X - Z, -15X	-10X, -10X

- Suppose prisoners may take action to alter the payoff structure by imposing penalties on informers who thwart escape attempts.
  - They modify the payoff matrix so that choosing to inform when the other player chooses to cooperate has a payoff of -2X – Z, where Z is some unspecified but very harsh penalty imposed on informers by other inmates.

- Cooperative equilibrium is the same as before
- If (-2X – Z) < -5X, then there is no longer a dominant strategy equilibrium in inform.
- Q. Are there Nash equilibria in pure strategies?
- Consider cooperate-cooperate? Is there incentive to defect here?
  - No! So, cooperate-cooperate is a Nash equilibrium…and, thus, it is clear that inform-cooperate or cooperate-inform cannot be a Nash.
- What about inform-inform?
  - It is a Nash equilibrium as well. No incentive to defect.

Prisoner's Dilemma Facing Two Convicts with Inmate Action Against Informers

- The inmate response to informing has produced a second Nash equilibrium in which there are coordinated escapes.
- Does this make prison management happy?
  - Probably not! And, they routinely take steps to remove informants from the population so that inmates cannot enforce the –Z punishment.
- This illustrates an important use of enforcement games.
  - They help us understand the behavior of agents, but can also be used to design incentive schemes so that desired outcomes are reached
  - This simple game provides a rationale for the behavioral responses of inmates and of prison management to the possibilities of escape.

Reckless Driver Game

- Drivers travel a road and experience a welfare gain (normalized to zero) from their trip if they travel at the speed limit.
  - If they can speed, welfare equals $w > 0$.
    - This is a net pvt gain in the welfare of drivers after all costs associated with speeding are subtracted.
    - Assume w is measured net of any costs due to liability for damages to other drivers.
  - Drivers caught speeding pay a fine f
  - Net gain from speed is equal to $(w - f)$
- Now, consider the police department.
  - Patrolling the highway regularly imposes a cost, p, on the police department.
  - If police patrol regularly, they will be able to catch all reckless drivers who are speeding on the highway.

Reckless Driver Game

- Reckless driving results in externalities for other motorists.
  - Assume the externality per reckless driver is fixed and equal to e.
- Police are rational and consider the costs of patrolling, the gains from fines, and the externalities imposed on the public.
  - They consider externalities because the police react to a political system that responds to the drivers.
- Police payoff is therefore equal to $(f - p - e)$ if they patrol and catch a speeding motorist, $-e$ if they don't patrol and motorists speed, $-p$ if they patrol and there are no speeders, and 0 if they do not patrol and there are no speeders.
- In summary, the problem has four crucial parameters that determine payoffs:
  - (1) w = net private gain to drivers from offending
  - (2) f = fine
  - (3) p = patrol cost
  - (4) e = externality cost of reckless driving

Reckless Driver Game

Driver's Choices

Enforcement Choices

	Reckless	Careful
Patrol	$(f-p-e), (w-f)$	$-p$, 0
Not Patrol	$-e$, w	0, 0

- Q. What is the cooperative equilibrium?
- Ans. In these types of enforcement games, the cooperative equilibrium is usually never found where enforcement costs are incurred.
  - Joint payoff if police patrol and there is speeding is $(w-p-e)$.
  - Joint payoff if police patrol and there is no speeding is $-p$.
  - Joint payoff if no patrol and there is speeding is $(w-e)$, which must be larger than $(w-p-e)$.
  - Joint payoff if no patrol and no speeding is 0, which is larger than $-p$.
  - So, the cooperative equilibrium is either Not Patrol-Reckless or Not Patrol-Careful, depending on the size of the externality.

Reckless Driver Game

		Driver's Choices	
		Reckless	Careful
Enforcement Choices	Patrol	$(f-p-e), (w-f)$	$-p$, 0
	Not Patrol	$-e$, w	0, 0

- Q. What is the dominant strategy for the driver?
- Ans. Depends on the sign of $(w-f)$. If $(w-f) > 0$, then driver has a dominant strategy (i.e., speed).
- Q. What about for the police?
- Ans. Depends on the sign of $(f-p)$. If $(f-p) < 0$, then police have a dominant strategy (i.e., do not patrol).
- Q. What about a Nash equilibrium in pure strategies?
- Ans. This depends on whether or not the driver and police have dominant strategies (i.e., if $(w-f) > 0$ and $(f-p) < 0$). Otherwise there may be no Nash in pure strategies.

Reckless Driver Game

Driver's Choices

Enforcement Choices

	Reckless	Careful
Patrol	$(f - p - e), (w - f)$	$-p$, 0
Not Patrol	$-e$, w	0, 0

- Criminal justice policy implications
  - 1.) If $(w - e) > 0$, then the cooperative equilibrium is for drivers to speed and the police to allow them to do it.
    - Does this mean that highways should not have speed limits?
      - Indeed, in some parts of the world, this is the policy, and it may well be welfare maximizing.
    - We can also think of this condition as a guide for setting speed limits on highways.
    - For instance, if $(w - e) > 0$ and there is a speed limit, the limit is too low and should be raised. The limit should be raised until $(w - e)$ is significantly below zero.
  - 2.) The condition that $(w - f) > 0$ results in a dominant strategy in speeding for the driver also has policy implications
    - The fine should be set high enough so that it can deter offenses under the criminal law.
    - Otherwise, the law becomes a form of taxation...and, it would be more efficient to simply collect a fee from drivers who intend to speed before they make a trip.

Reckless Driver Game

	Driver's Choices	
Enforcement Choices	Reckless	Careful
Patrol	$(f - p - e), (w - f)$	$-p$, 0
Not Patrol	$-e$, w	0, 0

- Criminal justice policy implications
  - 3.) The police have a dominant strategy in not patrolling if $(f - p) < 0$.
    - This implies that if the fine is below the cost of patrolling, the criminal justice system should not enforce the law.
    - Given the payoff function of the police, it is not reasonable to expect them to enforce laws that have small penalties and large enforcement costs.
  - From 2.) and 3.), the two conditions regarding dominant strategies suggest that fines should be higher than both w and p if activities are to be criminalized and laws are to be enforced.

Reckless Driver Game

Driver's Choices

Enforcement Choices		Reckless	Careful
	Patrol	$(f - p - e), (w - f)$	$-p$, 0
	Not Patrol	$-e$, w	0, 0

- Based on discussion above, let's assume the speed limit has been set sufficiently high so that $(w - e) < 0$ and, hence, the cooperative equilibrium is at Not Patrol – Careful.
- Furthermore, the fine is set greater than either w or p, so there is no dominant strategy for either player.
- Q. Is there a Nash equilibrium? Let's check…
  - If we are in the Patrol-Reckless cell, is there an incentive to defect for either party?
    - Yes, the driver will defect to Careful.
    - Now, if we are in the Patrol-Careful, what will happen?
      - The police have an incentive to defect to Not Patrol.
    - But, if we are in Not Patrol-Careful, then the drivers will have an incentive to speed…and so on and so forth.
  - This is a classic enforcement cycle where there is no Nash equilibrium.